1. Let $R$ be the finite region bounded by the graphs of $y = 3 \sin(x)$, $y = 6$, and $x = \pi$. Set up, but do not evaluate, definite integrals which represent the following quantities. Integrate with respect to $x$.

(a) The area of the region, $R$.

$$A = \int_{x_{\text{min}}}^{x_{\text{max}}} (Y_{\text{top}} - Y_{\text{botom}}) dx = \int_0^\pi (6 - 3 \sin(x)) dx$$

(b) The volume of the solid formed when $R$ is revolved around the line $y = 8$.

$$V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{(cross-sectional area)} dx = \int_0^\pi ((r_{\text{out}})^2 - (r_{\text{in}})^2) dx$$

$$V = \int_0^\pi ((8 - 3 \sin(x))^2 - (8 - 6)^2) dx$$

(c) The volume of the solid formed when $R$ is revolved around the line $x = -2$.

$$V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{(surface area)} dx = \int_0^\pi 2\pi r * h dx = \int_0^\pi 2\pi (x + 2) * (6 - 3 \sin(x)) dx$$
2. Evaluate the following indefinite integral:

\[ \int \frac{\sin^2(x)}{\sec(x) \csc^4(x)} \, dx \]

\[ \int \frac{\sin^2(x)}{\cos(x) \sin^4(x)} \, dx = \int \sin^6(x) \cos(x) \, dx \]

Need to use u-sub for this problem: \((u = \sin(x))\) and \((du = \cos(x) \, dx)\).

\[ \int u^6 \, du = \frac{1}{7} u^7 + C = \frac{1}{7} \sin^7(x) + C \]

3. Find the average value of the function below on the interval \([1, 9]\). Simplify.

\[ f(x) = \frac{8x}{x^2 + 9} \]

Average Value

\[ \left( \frac{1}{9 - 1} \right) \int_1^9 \frac{8x}{(x^2 + 9)} \, dx \]

Need to use u-sub and set \(u = x^2 + 9\) and \(4du = 8x \, dx\)

Average-value

\[ \frac{1}{8} \int_{10}^{90} \left( \frac{4}{u} \right) du = \frac{1}{2} \left( \ln(90) - \ln(10) \right) = \frac{1}{2} \ln(9) \]

4. Evaluate the indefinite integral:

\[ \int \frac{e^{9x}}{e^{18x} + 1} \, dx \]

Need to use u-sub to solve this problem: \(u = e^{9x}\) and \(\frac{1}{9} \, du = e^{9x} \, dx\)

\[ \int \frac{1}{u^2 + 1} \cdot \frac{1}{9} \, du = \frac{1}{9} \left( \arctan(u) \right) + C = \frac{1}{9} \left( \arctan(e^{9x}) \right) + C \]
5. At \( t \) hours, a population of bacteria is growing at a rate of

\[
r(t) = \frac{21e^{\frac{t}{2}}}{t^{\frac{3}{2}}} \text{ bacteria per hour}
\]

Compute the change in population size between times \( t = 169 \) s and \( t = 225 \) s. Simplify your answer.

Net change in population from \( t = 169 \) to \( t = 225 \) is defined as:

\[
\int_{169}^{225} r(t)dt = \int_{169}^{225} \left(\frac{21 * e^{\frac{t}{2}}}{t^{\frac{3}{2}}}\right) dt
\]

\( \rightarrow \) u-sub \( u = t^{\frac{1}{2}} \) and \( 2du = \left(\frac{1}{t^{\frac{3}{2}}}\right)dt \) and \( t = 169 \) equates to \( u = 13 \) and \( t = 225 \) equates to \( u = 15 \)

\[
\int_{13}^{15} (21 * 2 * e^{u})du = 42 * e^{15} - 42 * e^{13} \text{ bacteria}
\]

6. Estimate the x-value for the point of intersection on the graphs of \( y = x^3 + 2x \) and \( y = 2x + 4 \) using Newton’s Model with an initial estimate of \( x_1 = 1 \). You should use this model two times in order to obtain estimates \( x_2 \) and \( x_3 \). Your final estimate should be written as a simplified fraction.

\[
x^3 + 2x = 2x + 4 \rightarrow x^3 - 4 = 0
\]

Let \( f(x) = x^3 - 4 \) and apply Newton’s Method to estimate a root of \( f(x) \)

\[
f'(x) = 3x^2 \rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

\[
x_1 = 1 \rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 2
\]

\[
x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{5}{3}
\]
7. Use a linear approximation to estimate

$$\ln \left( \frac{95}{100} \right)$$

Write your answer as either a simplified fraction or a decimal value.

$$f(x) = \ln(x)$$ so we need to find the tangent line at $$x = 1$$.

$$f'(x) = \frac{1}{x}$$ at $$f'(1) = 1$$

So line is: $$0 = 1 + b$$

$$b = -1$$ so $$L(x) = x - 1$$

At $$x = 1$$, $$f(x)$$ is approximately equal to $$L(x)$$ so $$L \left( \frac{95}{100} \right) = \frac{95}{100} - 1 = -\frac{1}{20} = -0.05$$

8. Express the definite integral as the limit of Riemann Sums. Don not evaluate the limit.

$$\int_{-3}^{5} x^2 e^{\sin(x)} dx$$

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \delta x \to \lim_{n \to \infty} \sum_{k=1}^{n} (x_k)^2 (\delta x) (e^{\sin(x)})$$

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left( -3 + \left( \frac{8k}{n} \right) \right)^2 \left( e^{\sin \left( -3 + \frac{8k}{n} \right)} \right) \left( \frac{8}{n} \right)$$

$$f(x) = x^2 * e^{\sin(x)}$$

$$\delta x = \frac{(b-a)}{n} = \frac{8}{n}$$

$$x = a + k \delta x = -3 + \left( \frac{8}{n} \right) k$$

9. Fill in the missing information for the following two theorems.

(a) Continuous, differentiable, $$f'(c) = \frac{f(b)-f(a)}{(b-a)}$$

(b) Continuous, differentiable, $$f(a) = f(b) = f'(c) = 0$$
10. Some of the values of a polynomial \( f(x) \) are shown below in the table. If \( g(x) = 8xf'(x^2) \), then find the average value of \( g(x) \) on the interval [0, 2]. Simplify your answer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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<tr>
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<td>5</td>
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<td>8</td>
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<td>144</td>
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<tr>
<td>9</td>
<td>233</td>
</tr>
</tbody>
</table>

Average-value
\[
\left( \frac{1}{2-0} \right) \int_0^2 g(x)dx = \frac{1}{2} \int_0^2 8x \cdot f'(x)dx
\]

\( \rightarrow \) use u-sub where \( u = x^2 \) and \( 4du = 8xdx \)

Average-value
\[
\frac{1}{2} \int_0^4 4f'(u)du = 2(f(4) - f(0)) = 36
\]