The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

1. Let R be the finite region bounded by the graphs of $y = 3 \sin(x)$, $y = 6$, and $x = \pi$. Set up, but do not evaluate, definite integrals which represent the following quantities. Integrate with respect to x.

   (a) The area of the region, R.
   (b) The volume of the solid formed when R is revolved around the line $y = 8$.
   (c) The volume of the solid formed when R is revolved around the line $x = -2$.

2. Evaluate the following indefinite integral:

   $$\int \frac{\sin^2(x)}{\sec(x) \csc^4(x)} \, dx$$

3. Find the average value of the function below on the interval $[1, 9]$. Simplify.

   $$f(x) = \frac{8x}{x^2 + 9}$$
4. Evaluate the indefinite integral:

\[ \int \frac{e^{9x}}{e^{18x} + 1} \, dx \]

5. At \( t \) hours, a population of bacteria is growing at a rate of

\[ r(t) = \frac{12e^{t^5}}{t^5} \] bacteria per hour

Compute the change in population size between times \( t = 169 \) s and \( t = 225 \) s. Simplify your answer.

6. Estimate the x-value for the point of intersection on the graphs of \( y = x^3 + 2x \) and \( y = 2x + 4 \) using Newton’s Model with an initial estimate of \( x_1 = 1 \). You should use this model two times in order to obtain estimates \( x_2 \) and \( x_3 \). Your final estimate should be written as a simplified fraction.
7. Use a linear approximation to estimate

\[ \ln \left( \frac{95}{100} \right) \]

Write your answer as either a simplified fraction or a decimal value.

8. Express the definite integral as the limit of Riemann Sums. Don not evaluate the limit.

\[ \int_{-3}^{5} x^2 e^{\sin(x)} \, dx \]

9. Fill in the missing information for the following two theorems.

(a) **Mean Value Theorem** Let \( f \) be a function that satisfies the following two hypotheses:

(1) \( f \) is ____________________________ on the closed interval \([a, b] \)

(2) \( f \) is ____________________________ on the open interval \((a, b) \)

Then there is a number \( c \) in \((a, b) \) such that ____________________________
(b) **Rolle’s Theorem** Let $f$ be a function that satisfies the following three hypotheses

1. $f$ is __________________________ on the closed interval $[a, b]$
2. $f$ is __________________________ on the open interval $(a, b)$
3. ______________________________________________________________________

Then there is a number $c$ in $(a, b)$ such that __________________________

10. Some of the values of a polynomial $f(x)$ are shown below in the table. If $g(x) = 8xf'(x^2)$, then find the average value of $g(x)$ on the interval $[0, 2]$. Simplify your answer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
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