1. Let $R$ be the finite region bounded by the graphs of $y = 3\sin(x)$, $y = 6$, and $x = \pi$. Set up, but do not evaluate, definite integrals which represent the following quantities. Integrate with respect to $x$.

(a) The area of the region, $R$.

$$A = \int_{x_{\text{min}}}^{x_{\text{max}}} (Y_{\text{top}} - Y_{\text{botom}}) \, dx = \int_{0}^{\pi} (6 - 3\sin(x)) \, dx$$

(b) The volume of the solid formed when $R$ is revolved around the line $y = 8$.

$$V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{(cross-sectional area)} \, dx = \int_{0}^{\pi} ((r_{\text{out}})^2 - (r_{\text{in}})^2) \, dx$$

$$V = \int_{0}^{\pi} ((8 - 3\sin(x))^2 - (8)^2) \, dx$$

(c) The volume of the solid formed when $R$ is revolved around the line $x = -2$.

$$V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{(surface area)} \, dx = \int_{0}^{\pi} 2\pi r h \, dx = \int_{0}^{\pi} 2\pi (x + 2) \cdot (6 - 3\sin(x)) \, dx$$
2. Evaluate the following indefinite integral:

\[ \int \frac{\sin^2(x)}{\sec(x) \csc^4(x)} \, dx \]

\[ \int \frac{\sin(x)}{\cos(x) \ast \frac{1}{\sin^4(x)}} \, dx = \int \sin^6(x) \cos(x) \, dx \]

Need to use u-sub for this problem: \( u = \sin(x) \) and \( du = \cos(x) \, dx \).

\[ \int u^6 \, du = \frac{1}{7} u^7 + C = \frac{1}{7} \ast \sin^7(x) + C \]

3. Find the average value of the function below on the interval \([1, 9]\). Simplify.

\[ f(x) = \frac{8x}{x^2 + 9} \]

Average Value

\[ \left( \frac{1}{9 - 1} \right) \ast \int_1^9 \frac{8x}{(x^2 + 9)} \, dx \]

Need to use u-sub and set \( u = x^2 + 9 \) and \( 4du = 8 \, dx \)

Average-value

\[ \frac{1}{8} \int_{10}^{90} \left( \frac{4}{u} \right) \, du = \frac{1}{2} (\ln(90) - \ln(10)) = \frac{1}{2} \ln(9) \]

4. Evaluate the indefinite integral:

\[ \int \frac{e^{9x}}{e^{18x} + 1} \, dx \]

Need to use u-sub to solve this problem: \( u = e^{9x} \) and

\[ \frac{1}{9} \, du = e^{9x} \, dx \]

\[ \int \frac{1}{u^2 + 1} \ast \frac{1}{9} \, du = \frac{1}{9} (\arctan(u)) + C = \frac{1}{9} (\arctan(e^{9x})) + C \]
5. At $t$ hours, a population of bacteria is growing at a rate of

$$r(t) = \frac{12e^{t^5}}{t^5} \text{ bacteria per hour}$$

Compute the change in population size between times $t = 169$ s and $t = 225$ s. Simplify your answer.

Net change in population from $t = 169$ to $t = 225$ is defined as:

$$\int_{169}^{225} r(t)dt = \int_{169}^{225} \frac{(21 * e^{t^5})}{t^5} dt$$

$→ u$-sub $u = t^5$ and $2du = \left(\frac{1}{t^2}\right)dt$ and $t = 169$ equates to $u = 13$ and $t = 225$ equates to $u = 15$

$$\int_{13}^{15} (21 * 2 * e^u)du = 42 * e^{15} - 42 * e^{13} \text{ bacteria}$$

6. Estimate the x-value for the point of intersection on the graphs of $y = x^3 + 2x$ and $y = 2x + 4$ using Newton’s Model with an initial estimate of $x_1 = 1$. You should use this model two times in order to obtain estimates $x_2$ and $x_3$. Your final estimate should be written as a simplified fraction.

$$x^3 + 2x = 2x + 4 \rightarrow x^3 - 4 = 0$$

Let $f(x) = x^3 - 4$ and apply Newton’s Method to estimate a root of $f(x)$

$$f'(x) = 3x^2 \rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1 \rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 2$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{5}{3}$$
7. Use a linear approximation to estimate
\[ \ln \left( \frac{95}{100} \right) \]

Write your answer as either a simplified fraction or a decimal value.
\[ f(x) = \ln(x) \] so we need to find the tangent line at \( x = 1 \).

\[ f'(x) = \frac{1}{x} \ \text{at} \ f'(1) = 1 \]

So line is: \( 0 = 1 + b \)

\( b = -1 \) so \( L(x) = x - 1 \)

At \( x = 1 \), \( f(x) \) is approximately equal to \( L(x) \) so \( L\left( \frac{95}{100} \right) = \frac{95}{100} - 1 = \frac{-1}{20} = -0.05 \)

8. Express the definite integral as the limit of Riemann Sums. Don not evaluate the limit.
\[ \int_{-3}^{5} x^2 e^{\sin(x)} \, dx \]

\[ \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \delta x \to \lim_{n \to \infty} \sum_{k=1}^{n} (x_k)^2 (\delta x) (e^{\sin(x)}) \]

\[ \lim_{n \to \infty} \sum_{k=1}^{n} \left(-3 + \left( \frac{8k}{n} \right) \right)^2 \left( e^{\sin \left(-3+\frac{8k}{n} \right)} \right) \left( \frac{8}{n} \right) \]

\[ f(x) = x^2 \cdot e^{\sin(x)} \]

\[ \delta x = \frac{(b - a)}{n} = \frac{8}{n} \]

\[ x = a + k \cdot (\delta x) = -3 + \left( \frac{8}{n} \right) k \]

9. Fill in the missing information for the following two theorems.
(a) Continuous, differentiable, \( f'(c) = \frac{f(b) - f(a)}{(b-a)} \)

(b) continuous, differentiable, \( f(a) = f(b) = f'(c) = 0 \)
10. Some of the values of a polynomial \( f(x) \) are shown below in the table. If \( g(x) = 8xf'(x^2) \), then find the average value of \( g(x) \) on the interval \([0, 2]\). Simplify your answer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
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<tr>
<td>0</td>
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<tr>
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<td>144</td>
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<tr>
<td>9</td>
<td>233</td>
</tr>
</tbody>
</table>

Average-value

\[
\left( \frac{1}{2 - 0} \right) \int_{0}^{2} g(x) \, dx = \frac{1}{2} \int_{0}^{2} 8x \cdot f'(x) \, dx
\]

\[\rightarrow \text{use u-sub where } u = x^2 \text{ and } 4du = 8xdx\]

Average-value

\[
\frac{1}{2} \int_{0}^{4} 4f'(u) \, du = 2(f(4) - f(0)) = 36
\]