The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Apr 25, 6-7:30pm Gabby and Amy       Session 2: Apr 26, 6:30-8pm Chris and Elena

Can’t make it to a session? Here’s our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!
Multiple Choice Questions

1. Consider the initial value problem

\[(\cos t)y''' + (t - 1)y'' - \frac{1}{t + 1} y = t^3 - 1 \quad y'''(0) = 5, y'(0) = 0, y(0) = \pi\]

On what interval is this problem guaranteed to have a unique solution?

A) \((\frac{\pi}{2}, 1)\)
B) \((-1, \frac{\pi}{2})\)
C) \((-1, 1)\)
D) \((-\infty, \infty)\)

2. Find the steady-state solution of the heat equation \(4u_{xx} = u_t, \ u_x(0, t) = \frac{1}{2}u(0, t), \ u(3, t) = 15\)

A) DNE
B) 15
C) 6 + 3x
D) 15 \(−\) 4e\(^{-t}\)

3. Which of the following sets of functions are linearly INDEPENDENT?

(I) \(f(t) = 6t \quad g(t) = 3t^2 - 3 \quad h(t) = 3t - 6\)
(II) \(f(t) = 5t - 20 \quad g(t) = 5t + 15 \quad h(t) = 10t - 5\)
(III) \(f(t) = 2 \quad g(t) = 2t \quad h(t) = 2t^2\)

A) (I) and (III) \quad B) (III) only \quad C) (I) only \quad D) (II) only
4. The function \( f \) is defined on \([0,1)\) by \( f(x) = x^2 \). Which of the plots below represents the odd 2-periodic extension of \( f \)?

A) \( f(x) \)  
B) \( f(x) \)  
C) \( f(x) \)  
D) \( f(x) \)

5. The temperature \( u(x,t) \) in a bar of length 4 with heat diffusivity \( \alpha^2 = \frac{1}{4} \) satisfies the heat equation \( \alpha^2 u_{xx} = u_t \). If both ends of the bar are insulated and \( u(x,0) = 6 \) for \( 0 \leq x \leq 1 \) and 2 for \( 1 < x \leq 4 \), then evaluate the limit as \( t \to \infty \) for \( u(x,t) \)

A) 2  
B) 6  
C) 3  
D) 0  
E) 4

6. Which of the following integrals are zero for all \( L > 0 \)?

(I) \( \int_{-L}^{L} \sin(3t)e^{t^2} \, dt \)  
(II) \( \int_{-L}^{L} x^3 \cos x \, dx \)  
(III) \( \int_{-L}^{L} t \sin(t^4) \, dt \)  
(IV) \( \int_{-L}^{L} |x^2 - 5| \sin x \, dx \)

A) (I) only  
B) All  
C) (IV) only  
D) (II) only  
E) (III) only
7. Functions $f, g, h$ and $k$ are 6-periodic. Their values on $[-3,3)$ are given below. For which of these functions does the Fourier series converge at $x = 0$ to the value 1?

$$f(x) = \begin{cases} 
2 + x & -3 \leq x < 0 \\
1 & x = 0 \\
-2 & 0 < x < 3 
\end{cases}$$

$$g(x) = \begin{cases} 
1 + x & -3 \leq x < 0 \\
4 & x = 0 \\
2 - x^2 & 0 < x < 3 
\end{cases}$$

$$h(x) = \begin{cases} 
x^2 - 1 & -3 \leq x < 0 \\
-1 & x = 0 \\
3 & 0 \leq x < 3 
\end{cases}$$

$$k(x) = \begin{cases} 
3 + x & -3 \leq x < 1 \\
1 & x = 1 \\
x - 8 & 1 < x < 3 
\end{cases}$$

A) $f$
B) $h$
C) None
D) $g$ and $k$
E) $f$ and $h$

Free Response Questions

8. Compute all the eigenvalues and corresponding eigenfunctions for the boundary value problem

$$y'' - \lambda y = 0 \quad y'(-2) = 0, y(0) = 0$$

If a certain range of the real numbers does not include any eigenvalues, show why there are none in that range.
9. Consider the function $f(x) = 1 - x$ defined on the interval $x \in [-1, 1)$

(a) Sketch the 2-periodic extension of $f(x)$ on the interval $x \in [-3, 3]$

(b) Compute the 2-periodic Fourier series representation of $f(x)$
10. Find the general solution of

\[ y^{(4)} - 2y'' + y = e^t + t^2 \]

(Hint: Use Method of Undetermined Coefficients)