The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb 28, 4-5:30pm Jay and Greg  
Session 2: None

Can’t make it to a session? Here’s our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!
There are some questions on this worksheet that will not be covered until the final. If you are using this worksheet to prepare for Quiz 2, please do problems 2, 3, 5, 6, 10, 13, 14, 16, 17, 23, and 24. Worksheet 1 problems 1, 6, 8, and 14 will also be covered on Quiz 2.

Concepts

1. A potential energy function \( U(x) \) (in eV) is plotted with respect to position (blue) over the interval \( 1 \leq x \leq 5 \). \( U(x) = 0 \) everywhere else. A particle with a total energy of 3 eV is allowed to interact with this potential (orange). Assume (for now) that this system obeys classical behavior.

a) Highlight the interval(s) of the classically forbidden region.

b) Determine the behavior of the particle on \( 2 \leq x \leq 3.5 \)

c) Where can the particle be placed so that it will remain at rest?

d) Suppose the particle is initially placed at \( x = 4 \). Describe its behavior as \( t \rightarrow \infty \)

e) What is the minimum total energy required such that the particle will escape to infinity?
2. Consider the wavefunction $\Psi(x) = Ne^{i(kx-\omega t)}$

a) What is the momentum of a particle given by $\Psi(x)$?

b) Show that the probability density, as a function of position, is constant in time.

c) Knowing that this wavefunction has definite momentum (momentum eigenstate), what can be said of its position?

3. **True or False**: A wavefunction of a superposition of momentum eigenstates has more position certainty than a wavefunction with only one momentum eigenstate.

4. According to the band structure model, which of the following materials has the largest and smallest energy gap? Aluminum (good conductor), silicon (semiconductor), and silicon dioxide glass (good insulator).

5. Consider a situation with a particle trapped in an infinite square well on $0 \leq x \leq L$ where all the particle’s energy is kinetic

a) What must the potential energy be on $-\infty \leq x \leq 0$?

b) What must the potential energy be on $0 < x < L$?

c) What must the potential energy be on $L \leq x \leq \infty$

d) Give the general form of $\Psi(x)$ for a particle in the square well.

e) What is the value of $\Psi(x)$ on the boundaries of the square well?
6. If the ground state of an infinite square well has energy equal to $E_1$ then which state will have energy equal to $9E_1$?

7. Consider the following two quantum harmonic oscillators:

Oscillator 1: Particles with constant $k$ and mass $m$
Oscillator 2: Particles with constant $k$ and mass $2m$

Let $E_n = (n + \frac{1}{2}) \frac{\hbar}{2\pi} \sqrt{\frac{k}{m}}$ determine the allowed energies absorbed by the oscillators. Which oscillator is more likely to absorb low frequency light?

8. Consider two different oscillators:

Oscillator 1: Particles with constant $2k$ and mass $\frac{1}{2}m$
Oscillator 2: Particles with constant $4k$ and mass $m$

Let $E_n = (n + \frac{1}{2}) \frac{\hbar}{2\pi} \sqrt{\frac{k}{m}}$ determine the allowed energies absorbed by the oscillators. Which oscillator is more likely to absorb low frequency light?

9. QM tells us that particles can penetrate into the classically forbidden region, although the wavefunctions for such cases resemble exponential decay. The decay constant is determined by the mass of the particle involved and the energy difference between the particle and the forbidden region. Is this constant higher for heavier or lighter particles?
10. A particle beginning in an energy eigenstate has __________ probability density with time, while particle beginning in a superposition of energy eigenstates have a probability density that is __________ in time.

11. **True or False:** It is possible for the wavefunction of a particle to depend on time, but the probability density to be time independent.

12. Suppose in a double-well system, at time t = 0 the wavefunctions associated with the first two energy eigenstates interfere constructively in the rightmost well, and destructively in the leftmost well. In which well are we more likely to find the particle initially? Give an example wavefunction where this is true.

**Mathematical Techniques**

13. Verify the following identity: \( \sin(kx) = \frac{(e^{ikx} - e^{-ikx})}{2i} \)

14. Derive de Broglie’s relation. We’ll get you going:
   a) What is Einstein’s famous Mass-Energy relation? (Hint: E \( \propto \) m by some factor squared)
   b) Replace the constant of proportionality with v (for any arbitrary velocity)
   c) Equate this with Planck’s equation: E = hf
   d) Simplify

15. Show that \( \Psi(x) = Ne^{-\alpha x^2} \) is an energy eigenstate of quantum harmonic oscillators by showing that it’s a solution to the time-independent Schrödinger Equation.
16. A wavefunction given by $\Psi(x) = C_1\Psi_1 + C_2\Psi_2 + C_3\Psi_3$ is a superposition of three eigenstates. Let $C_1 = C_2 = 0.5$. What must $C_3$ be for $\Psi(x)$ to be normalized?

17. Compare the following integrals:

$$\int_0^5 N^2\sin^2\left(\frac{8\pi x}{5}\right)dx$$

$$\int_0^5 N^2\sin^2\left(\frac{\pi x}{5}\right)$$

where $N = \sqrt{\frac{2}{5}}$

If you are computing these integrals, you have missed the point of this question. (Hint: These integrands are the probability densities of the infinite square wells at energy levels 1 and 8).

18. Suppose two particles in a quantum harmonic oscillator have different values of $k$, but identical mass. Which graph corresponds to the particle with lower $k$? Orange or Blue?

![Graph showing comparison of two wave functions]

19. Show that the formula for the period of oscillation between eigenstates can be rewritten as:

$$T = \frac{2\pi}{\omega_1 - \omega_2} = \frac{h}{E_1 - E_2}$$
20. Suppose $\Psi_1(x,t=0)$ and $\Psi_2(x,t=0)$ are energy eigenstates to a quantum system with energy $E_1$, $E_2$, and one particle starts in state $\Psi_1$ at $t = 0$. How does its wavefunction evolve with time? How about its probability density?

21. If the particle is NOT in an energy eigenstate, but a superposition of two energy eigenstates $\Psi(x,t=0) = (\Psi_1(x,t=0) + \Psi_2(x,t=0))$, how does its wavefunction evolve with time? How about its probability density?

Calculations

22. The ground state energy of a quantum harmonic oscillator is 1.7 eV. Suppose an electron is in a superposition of the ground state and the first excited state and we know its probability density at $t = 0$. How long do we have to wait before its probability density returns to the same distribution at $t = 0$?
   a) $1.94 \times 10^{-16}$ s
   b) $3.87 \times 10^{-16}$ s
   c) $1.22 \times 10^{-15}$ s
   d) $2.44 \times 10^{-15}$ s
   e) $5.36 \times 10^{-15}$ s

23. An electron is at the second excited state in an 1D infinite square well and decays into the ground state while giving off a photon. If the width of the well is measured to be 1 nm, what is the wavelength of the emitted photon?
   (a) 220 nm
   (b) 413 nm
   (c) 675 nm
   (d) 928 nm
   (e) $1.10 \mu$m
24. A beam of electrons is prepared having a deBroglie wavelength of 0.05 nm. What is the kinetic energy of the beam?

a) 37.5 eV  
b) 601.7 eV  
c) 223.6 eV  
d) 454.9 eV  
e) 524.5 eV

25. Consider the oscillator below:

\[\begin{array}{c}
11 \text{ eV} \\
9 \text{ eV} \\
7 \text{ eV} \\
5 \text{ eV} \\
3 \text{ eV} \\
1 \text{ eV}
\end{array}\]

Six electrons are in a harmonic oscillator. Fill in the diagram to represent the oscillator’s lowest excited state, using arrows to represent the spin of the electrons. What would the diagram look like if it is irradiated with light of energy 3 eV?
26. A particle is in the ground state of a quantum harmonic oscillator (ground state energy $= \frac{1}{2} \hbar \omega$), what would happen if some scientists shoot a photon with energy $\hbar \omega$ at it? What if they change the photon energy to $0.9\hbar \omega$, $1.1\hbar \omega$?