The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: 3/28, 4:30-6pm Jesse and Gabby
Session 2: 3/29, 6:30-8pm Rehnuma and Amy

Can’t make it to a session? Here’s our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!
1. Rewrite the function $y(t) = 4 \cos(7t) - 4 \sin(7t)$ in phase amplitude.
   
   A) $4\sqrt{2} \cos(7t + \frac{\pi}{4})$
   
   B) $4\sqrt{2} \cos(7t - \frac{\pi}{4})$
   
   C) $4 \cos(7t + \frac{\pi}{4})$
   
   D) $4\sqrt{2} \cos(7t)$

2. The differential equation $mu'' + 4u' + 8u = 0$ describes a mass-spring system. For what values of $m$ is the system underdamped?
   
   A) $m < \frac{1}{2}$
   
   B) $m < 2$
   
   C) $m > \frac{1}{2}$
   
   D) $m > 2$

3. For what forcing frequency $\omega$ is it possible for the solution to $18y'' + 2y = 81 \cos(\omega t)$ to grow without bound?
   
   A) $\omega = 3$
   
   B) $\omega = 9$
   
   C) $\omega = \frac{1}{3}$
   
   D) $\omega = \frac{1}{9}$
4. Consider the following statements

(i) If \( Y_1 \) is a particular solution to the DE

\[ y'' + p(t)y' + q(t)y = g(t) \]

then \( Y_2 = cY_1 \) is also a solution to the differential equation, where \( c \) is a constant.

(ii) If \( y_1 \) and \( y_2 \) are solutions to

\[ ay'' + by' + cy = g(t) \]

then \( y_1 - y_2 \) is a solution to the homogeneous differential equation \( ay'' + by' + cy = 0 \) where \( a, b, \) and \( c \) are constants.

(iii) The solution to

\[ ay'' + by' + cy = 0 \]

with initial conditions \( y(0) = A \) and \( y'(0) = B \), is unique on \( t \in (-\infty, \infty) \) where \( a, b, \) and \( c \) are constants.

(iv) Two solutions to a linear second order ODE can cross each other.

Which of the above statements are always true?

A) (i), (iii) and (iv)
B) (iii) and (iv)
C) (ii) and (iii)
D) (ii), (iii) and (iv)

5. Consider the following initial value problem \( (t-5)y'' + \csc(t)y' + y = e^t, \ y'(4) = 1 \) and \( y(4) = \pi \).

What is the largest interval on which the initial value problem is guaranteed to exist?

A) \((0,5)\)
B) \((0, 2\pi)\)
C) \((\pi, 5)\)
D) \((\pi, 2\pi)\)
6. Identify the form of the particular solution for $y'' - 16y = (t - 3)e^{-4t} + (4t + 3)\sin(2t)$

A) $(At + B)te^{-4t} + (Ct + D)t\sin(2t) + (Et + F)t\cos(2t)$

B) $(At + B)e^{-4t} + (Ct + D)\sin(2t) + (Et + F)\cos(2t)$

C) $(At + B)e^{-4t} + (Ct + D)t\sin(2t) + (Et + F)t\cos(2t)$

D) $(At + B)te^{-4t} + (Ct + D)\sin(2t) + (Et + F)\cos(2t)$

7. Find the general solution for the differential equation

$$y'' + 4y' + 4y = 0$$
8. Which of the following plots represents a solution to the ODE

\[ y'' + 7y' + 6y = 0 \]

A) (ii) only  
B) (iii) only  
C) (i) and (iv) only  
D) (ii) and (iii) only
9. We know that $y(t) = e^t$ is a solution to the ODE

$$2ty'' + (1 - 4t)y' + (2t - 1)y = 0$$

for $y > 0$. Find the general solution.