1. Find $f'(x)$ given that $f(x) = 4\sqrt{\arctan(x^9)}$

$$f(x) = 4\sqrt{\arctan(x^9)}$$

$$f(x) = (\arctan(x^9))^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-\frac{3}{4}} \cdot \frac{d}{dx}(\arctan(x^9))$$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-\frac{3}{4}} \cdot \left(\frac{1}{x^{18} + 1}\right) \cdot \frac{d}{dx}(x^9)$$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-\frac{3}{4}} \cdot \left(\frac{1}{x^{18} + 1}\right) \cdot 9x^8$$

2. Suppose that $A$ represents the number of grams of a radioactive substance at time $t$ seconds. Given that $\frac{dA}{dt} = -0.125A$, how long does it take 12 grams of the substance to be reduced to 4 grams?

First recall that $\frac{d}{dx} = ky$ so $y = ce^{kx}$. So $\frac{dA}{dt} = -0.125A$ and $A = ce^{-0.125t}$

Plugging in $A = 12$ when $t = 0$ gives us $12 = ce^{-0.125*0}$. Thus, $A = 12e^{-0.125t}$

$$4 = 12e^{-0.125t}$$

$$\ln\left(\frac{4}{12}\right) = -0.125t$$

$$t = \frac{\ln\left(\frac{1}{3}\right)}{-0.125} = 8 \ln(3)$$
\[ t = 8 \ln(3) \]
3. A streetlight is mounted at the top of a tall pole with $H = 16.5$ ft. Jennifer’s height is $h = 5.5$ ft tall. She walks away from the pole with a speed of 8 ft/s along a straight path. How quickly is the length of her shadow on the ground increasing when she is 15 ft from the pole?

Given: $\frac{dx}{dt} = 8$, we want $\frac{dy}{dx}\big|_{x=15}$

Use the below diagrams to help solve the problem

![Diagram showing the relationship between the pole, light, and shadow](image)

\[
\frac{X + Y}{H} = \frac{Y}{h} = X + Y = \frac{Y}{h} * H = 3Y
\]

\[X = 2Y\]

\[\frac{d}{dt}(X) = \frac{d}{dy}(2Y)\]

\[\frac{d}{dx} = 2 \frac{d}{dy}\]

\[8 = 2 \frac{d}{dy}\]

\[\frac{d}{dy} = 4\]

Therefore, the shadow length is increasing at a rate of 4 ft/s
4. The top of a ladder slides down a vertical wall at a rate of 8 m/s. At the moment when the bottom of the ladder is 4 meters from the wall, it slides away from the wall at a rate of 15 m/s. How long is the ladder?

Note that L is a constant length.

Given: \( \frac{du}{dt} = -8 \) and \( \frac{dx}{dt} \big|_{x=4} = 15 \)

We want L:

\[
X^2 + Y^2 = L^2
\]

\[
\frac{d}{dt}(X^2 + Y^2) = \frac{d}{dt}(L^2)
\]

\[
2X \frac{dx}{dt} + 2Y \frac{dy}{dt} = 0
\]

\[
2 \times 4 \times 15 + 2 \times Y \times (-8) = 0
\]

\[
Y = \frac{15}{2}
\]

\[
L = \sqrt{X^2 + Y^2}
\]

\[
L = 8.5 \text{ m}
\]
5. Find the absolute minimum y-value of the given function:

\[ y = \frac{2x}{\sqrt{x - 81}} \]

Domain of the function: \( x > 81 \).

\[ y' = \frac{2 \cdot \sqrt{x - 81} \cdot (-2x) \cdot (\frac{1}{2}) \cdot (x - 81)^{-\frac{1}{2}}}{(\sqrt{x - 81})^2} \]

\[ y' = \frac{2\sqrt{x - 81} - \frac{x}{\sqrt{x - 81}}}{x - 81} \]

\[ y' = \frac{2(x - 81) - x}{(x - 81)(\sqrt{x - 81})} \]

\[ y' = \frac{x - 162}{(x - 81)\sqrt{x - 81}} \]

There is an absolute max at \( x = 162 \) and that amount is 36, which can be found by plugging 162 into the function for \( y \).
6. A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown, but only has 600 feet of fencing available. Determine the values for x and y which will maximize the total area enclosed by these three pens.

\[ \text{Area} = 3xy \]
\[ \text{Perimeter} = 600 = 6x + 4y \]

\[ 4y = 600 - 6x \]
\[ y = 150 - 1.5x \]

Area is now equivalent to:

\[ \text{Area} = 3x \times (150 - 1.5x) \]
\[ \text{Area} = 450x - 4.5x^2 \]

Now, we must maximize A for x in the range of (0, 100)

\[ 0 = 450 - 9x \]
\[ 9x = 450 \]
\[ x = 50 \]

Check for the values of A':

\[ + + + \quad - - - \]
\[ 0 \quad 50 \quad 100 \]

So we can see that there is an absolute maximum at \( x = 50 \). Evaluate y at \( x = 50 \)

\[ y = 150 - 1.5 \times (50) \]
\[ y = 75 \]

\[ \text{Area} = 3 \times 50 \times 75 = 11,250 \text{ ft}^2 \]
7. Find \( \frac{dy}{dx} \) given the following:

\[
\sin(x^2 + y^3) = 5y + 8x
\]

It is okay to leave your answer in terms of both \( x \) and \( y \).

\[
\frac{d}{dx}(\sin(x^2 + y^3)) = \frac{d}{dx}(5y + 8x)
\]

\[
\cos(x^2 + y^3) \cdot (2x + 3y^2 \cdot \frac{dy}{dx}) = 5 \cdot \frac{dy}{dx} + 8
\]

\[
2x \cdot \cos(x^2 + y^3) + 3y^2 \cdot \frac{dy}{dx} \cdot \cos(x^2 + y^3) = 5 \cdot \frac{dy}{dx} + 8
\]

\[
(3y^2 \cos(x^2 + y^3) - 5) \frac{dy}{dx} = 8 - 2x \cos(x^2 + y^3)
\]

\[
\frac{dy}{dx} = \frac{8 - \cos(x^2 + y^3)}{3y^2 \cos(x^2 + y^3) - 5}
\]

8. Evaluate the following derivatives:

(a) \( \frac{d}{dx} \cos(x) \)

\[
\frac{d}{dx} \cos(x) = -\sin(x)
\]

(b) \( \frac{d}{dx} \csc(x) \)

\[
\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)
\]

(c) \( \frac{d}{dx} \tan(x) \)

\[
\frac{d}{dx} \tan(x) = \sec^2(x)
\]

(d) \( \frac{d}{dx} \arcsin(x) \)

\[
\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}
\]
(e) \[ \frac{d}{dx} \ln(x) \] 
\[ \frac{d}{dx} \ln(x) = \frac{1}{x} \]

9. A function \( f(x) \) has the first derivative \( f'(x) = e^{0.5x}(10x - 60) \)

(a) Upon which interval is \( f(x) \) increasing?

\[
\begin{array}{c|c|c}
\text{Values of } f'(x): & + & - \\
\hline
0 & 6 & \\
\end{array}
\]

Based off of this, we can say \( f \) is increasing on the interval \((6, \infty)\). So the answer is \([6, \infty)\).

(b) Upon which interval is the graph of \( f(x) \) concave down?

\[
f'(x) = 0.5e^{0.5x}(10x - 60) + e^{0.5x} \cdot 10
\]

\[
f'(x) = e^{0.5x}(0.5(10x - 60) + 10)\]

\[
f'(x) = e^{0.5x}(5x - 20)
\]

\[
\begin{array}{c|c|c}
\text{Values of } f'(x): & + & - \\
\hline
0 & 4 & \\
\end{array}
\]

The function is concave down on the interval \((-\infty, 4)\)
10. Evaluate each of the following limits:

(a) \[ \lim_{x \to \infty} \frac{2 \ln(x)}{\sqrt[3]{x}} \]

\[ \lim_{x \to \infty} \frac{2 \ln(x)}{\sqrt[3]{x}} = 2 \lim_{x \to \infty} \frac{\ln(x)}{\sqrt[3]{x}} \]

Using l’Hoptial:

\[ 2 \lim_{x \to \infty} \frac{1}{x} = 2 \lim_{x \to \infty} \frac{3}{x^{2/3}} = 2(0) = 0 \]

\[ \left( \text{Blue} = \sqrt[3]{x}, \text{orange} = 2 \ln(x) \right) \]

(b) \[ \lim_{x \to 0} \frac{e^{10x} - 1}{5x} \]

For this limit we must employ l’Hopital’s Rule

\[ \lim_{x \to 0} \frac{e^{10x} - 1}{5x} = \lim_{x \to 0} \frac{10e^{10x}}{5} = 2 \]
\[ \lim_{x \to \infty} \frac{e^{10x} - 1}{5x} \]

The numerator approaches infinitely much more rapidly than the denominator, thus the \( \lim_{x \to \infty} \to \infty \)

(Note about plots, these plots do not have the coefficients attached to \( x \) due to size constraints of the page. The general relation still holds and, in fact, is exacerbated by the coefficients)